NESA NATIONAL EXAMINATION AND SCHOOL INSPECTION AUTHORITY

Mathematics II

029

09/07/2025 8.30 AM-11.30 AM

Names: .....

Index number: .....

SUBJECT: MATHEMATICS II

COMBINATIONS:

- MATHEMATICS-CHEMISTRY-BIOLOGY **(MCB)**
- MATHEMATICS -COMPUTER SCIENCE-ECONOMICS **(MCE)**
- MATHEMATICS-ECONOMICS-GEOGRAPHY **(MEG)**
- MATHEMATICS -PHYSICS-COMPUTER SCIENCE **(MPC)**
- MATHEMATICS-PHYSICS-GEOGRAPHY **(MPG)**
- PHYSICS-CHEMISTRY-MATHEMATICS **(PCM)**

DURATION: 3 HOURS

S6 QUESTIONS and ANSWERS BOOKLET

ADVANCED LEVEL NATIONAL EXAMINATIONS, 2024-2025

Instructions to candidates:

1) Write your names and index number on the answer booklet as written on your registration form.
2) **DO NOT** open this question paper until you are told to do so.
3) This paper consists of **two** sections: **A** and **B**.

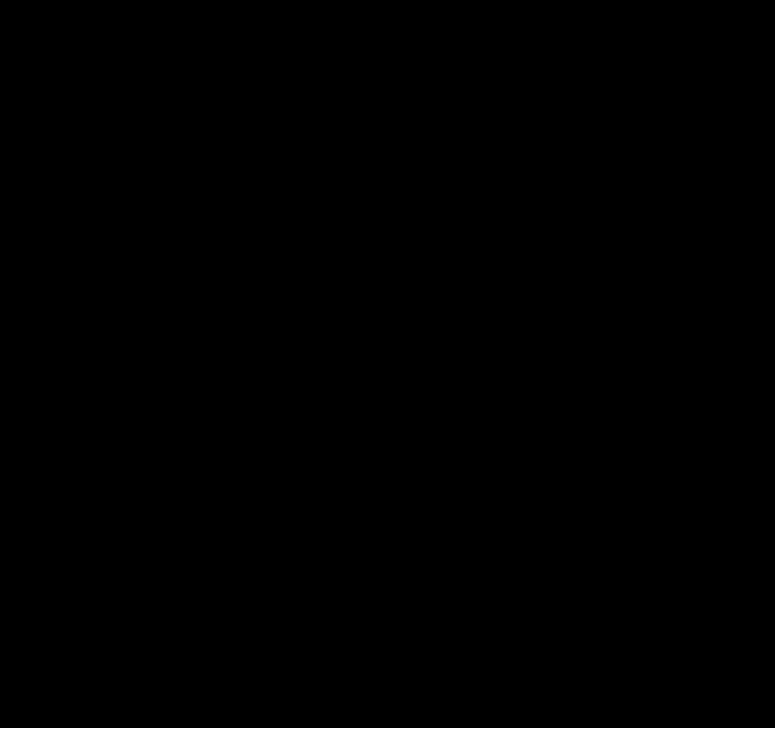
   Section A: Attempt **all** questions. **(55 marks)**

   Section B: Attempt **only three** questions. **(45 marks)**
4) **Geometrical instruments and silent non-programmable calculators may be used**.
5) Use only a **blue** or **black** pen.
6) For **multiple choice questions**, **circle** a letter corresponding to the correct answer.

TOTAL MARKS...../100

FOR EXAMINER'S USE ONLY

Questions	Marks	Questions	Marks
1		16	
2		17	
3		18	
4		19	
5		20	
6		21	
7		22	
8		23	
9		24	
10		25	
11		26	
12		27	
13		28	
14		29	
15		30	



Section A: Attempt all questions (55 marks).

1) State whether each of the following statements is **True** or **False**:

(3 marks)

a) The matrix $\begin{bmatrix} 2 & 1 \\ 0 & 4 \\ 5 & 2 \end{bmatrix}$ has dimensions 2 × 3.

b) The matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ is an identity matrix.

c) When solving a system $AX = B$ using inverse matrices, you use the equation: $X = b^{-1}A^{-1}$.

d) "Not invertible" is the same as "singular."

e) If determinant of matrix A different to zero then A is not invertible.

f) The function $f(x) = \cos x$ is a linear transformation from $\mathbb{R}$ into $\mathbb{R}$.

2) Use **True** or **False** for the following statements: if the differential equation is defined as $x\frac{dy}{dx} + \frac{3}{\frac{dy}{dx}} = y^2$: **(3 marks)**

a) The degree of the given differential equation is 1.
b) The order of the given differential equation is 2.
c) The given differential equation is linear differential equation.

3) Choose and circle the letter that corresponds to the right answer: By solving the inequality $|3-\log_2 x|<2$, we obtain: **(4 marks)**

a) $S=(2,\quad 32)$

b) $S=[2,\ 32]$

c) $S=[2,\quad 48]$

d) $S=[2,\quad \infty)$

4) Circle the letter that corresponds to the best answer.
The particular solution of the differential equation $\frac{d\theta}{dT}=2e^{3t-2\theta}$, given that $t=0$ when $\theta=0$ equals **(4 marks)**

a) $3e^{3\theta}=4e^{3t}-1$

b) $3e^{2\theta}=4e^{3t}-1$

c) $\frac{e^{3\theta}}{2}=\frac{2e^{2t}}{3}-\frac{1}{6}$

d) $\frac{e^{3\theta}}{2}=-\frac{2e^{3t}}{3}-\frac{1}{6}$

5) Work out and circle the letter related to the best answer: Consider three vectors in Euclidean space $\mathbf{R}^3$: $u = -3i + 4j + 12k$, $v = 4i - 3j + 2k$ and $w = mi + 3j - nk$

a) The values of m and n if $u = v \times w$ are equal to **(3 marks)**

(i) $\begin{cases} m = 0 \\ n = 1 \end{cases}$

(ii) $\begin{cases} m = 1 \\ n = 0 \end{cases}$

(iii) $\begin{cases} m = 4 \\ n = 12 \end{cases}$

(iv) $\begin{cases} m = 12 \\ n = 4 \end{cases}$

b) The area of the triangle $(0,\ v,\ w)$ is **(2 marks)**

(i) 4.5 *units area*

(ii) 6.5 *units area*

(iii) 9 *units area*

(iv) 3 *units area*

6) Work out and circle the letter that corresponds to the right answer.

Let us consider two complex numbers $z = \sqrt{3} + i$ and $w = 1 + i$

Multiply ***z*** and ***w*** in both Cartesian and polar forms and find that the value of $\cos\frac{5\pi}{12}$ and $\sin\frac{5\pi}{12}$ are

(3 marks)

a) $\frac{\sqrt{3}-1}{\sqrt{2}}$ *and* $\frac{\sqrt{3}+1}{\sqrt{2}}$ *respectively* .

b) $\frac{\sqrt{3}+1}{\sqrt{2}}$ *and* $\frac{\sqrt{3}-1}{\sqrt{2}}$ *respectively* .

c) $\frac{\sqrt{6}+\sqrt{2}}{4}$ *and* $\frac{\sqrt{6}-\sqrt{2}}{4}$ *respectively* .

d) $\frac{\sqrt{6}-\sqrt{2}}{4}$ *and* $\frac{\sqrt{6}+\sqrt{2}}{4}$ *respectively* .

7) Circle the best answer: A bag contains 6 red, 4 white and 8 blues balls. If 3 balls are drawn at random, the probability that one is red, one is white and one is blue equals. **(4 marks)**

a) $\frac{13}{24}$

b) $\frac{3}{18}$

c) $\frac{1}{192}$

d) $\frac{4}{17}$

8) If the system of equation is presented as $\begin{cases} e^x + e^y - 8 = 0 \\ 2e^x + e^{2y} - 16 = 0 \end{cases}$, circle the letter that is related to the best answer for the value of x and y: **(3 marks)**

a) $\begin{cases} x = \ln 2 \\ y = \ln 4 \end{cases}$

b) $\begin{cases} x = \ln 3 \\ y = \ln 6 \end{cases}$

c) $\begin{cases} x = \ln 5 \\ y = \ln 3 \end{cases}$

d) $\begin{cases} x = \ln 6 \\ y = \ln 2 \end{cases}$

9) The mean of 5 observations is 4.4 and the variance is 8.24. If three of the five observations are 1, 2 and 6. Find the other two. **(3 marks)**

10) Work out and circle the letter that corresponds to the best answer.

Given that $\overrightarrow{BA} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$, find $\overrightarrow{AC}$. **(3 marks)**

a) $\begin{pmatrix} -5 \\ 4 \end{pmatrix}$

b) $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$

c) $\begin{pmatrix} 4 \\ -5 \end{pmatrix}$

d) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

11) Circle the letter that corresponds to the right answer.

The equation of the line in symmetrical form is: **(3 marks)**

a) $\frac{x}{0}=\frac{y}{1}=\frac{z}{1}$

b) $\frac{x}{1}=\frac{y}{0}=\frac{z}{0}$

c) $\frac{x}{1}=\frac{y}{1}=\frac{z}{0}$

d) $\frac{x}{1}=\frac{y}{1}=\frac{z}{0}$

12) A surveyor observes that the angle of elevation of the top of a perpendicular building as19°. He moves 120 m near the building and finds the angle of elevation is now 47°. Determine the height of the building. **(4 marks)**

13) Use standard results on summations to find the value of:

$\sum_{r=36}^{48}(r-1)(3r-2)$ **(4 marks)**

14) Work out and circle the right answer.

The area contained between the curve $y = x^2 - 1$ and the x-axis is rotated about the x-axis through 360°. The volume of the solid generated is equal to: **(4 marks)**

a) $V = \frac{12\pi}{13}$ cubic units

b) $V = \frac{16\pi}{15}$ cubic units

c) $V = \frac{15\pi}{13}$ cubic units

d) $V = \frac{19\pi}{14}$ cubic units

15) Determine the exact values for $cosx$ and $sinx$ given that $\tan x = \frac{-12}{5}$ when $\frac{3\pi}{2} < x < 2\pi$. **(5 marks)**

Section B: Attempt any three questions only (45 marks)

Space in which to answer the three questions is at the end of this section.

16) Let us consider the following table:

x_i	-5	-1	3	10	13
y_i	33	25	17	3	-3

a) Calculate the coefficient of correlation and comment on it.

b) Find the equation of regression line y on x. **(10 marks)**

Estimate the value of x when $y=16$. **(5 marks)**

17) Let the matrix $A=\begin{bmatrix}2 & 2\\ 1 & 3\end{bmatrix}$

a) Find all eigenvalues and corresponding eigenvectors. **(6 marks)**

b) Find a nonsingular matrix P such that $D=P^{-1}AP$ is diagonal, and find P^{-1}. **(2 marks)**

c) Find A^6 and $f(A)$, where $f(t)=t^4-3t^3-6t^2+7t+3$. **(5 marks)**

d) Find a "real cube root" of B—that is, a matrix B such that $B^3 = A$ and B has real eigenvalues. **(2 marks)**

18) a) Find the equation of the locus of all points such that the difference of their distances from $(4,\ 0)$ and $(-4,\ 0)$ is always equal to 2.

(7 marks)

b) Consider the curve E of equation $E: 4x^2+9y^2-48x+72y=-144$.
Show that the curve E represents an ellipse. **(8 marks)**

19) Two sequences $\{U_n\}$ and $\{V_n\}$ over $\mathbb{N}_0$ are given by

$U_1=12,\ U_{n+1}=\dfrac{U_n+2V_n}{3}$ and $V_1=1,\ V_{n+1}=\dfrac{U_n+3V_n}{4}$ with $W_n=V_n-U_n$

a) Show that $\{W_n\}$ is a geometric sequence. **(3 marks)**

b) Express W_n in terms of n. **(4 marks)**

c) Show that $\{W_n\}$ is convergent and hence determine its limit. **(3 marks)**

d) Show that $\{U_n\}$ is decreasing and $\{V_n\}$ is increasing. **(5 marks)**

20) Consider the function $f(x) = \frac{2\ln x}{x}$

a) Find the domain of definition **(1 mark)**

b) Find the limits at the boundaries of the domain **(2 marks)**

c) Deduce the equations of asymptotes **(2 marks)**

d) Compute the first derivative and second derivative **(2 marks)**

e) Find the open intervals over which the graph of the given function is:

i) Increasing or decreasing **(2 marks)**

ii) Concave upward or downward **(2 marks)**

f) Determine the intersection(s) of the graph of the given function with axes of coordinates **(2 marks**

g) Sketch the graph **(2 marks)**

Space in which to answer the three chosen questions from section B.

-END-